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Title of the talk: A mean-field equation coming from a control problem with partial observation

Abstract: The starting point of the talk is a recent work with Juan Li (Shandong University, Weihai, P.R.China) and Jin Ma (University of South California, Los Angeles, U.S.A.), "*A mean-field stochastic control problem with partial observations*" (Annals of Appl.Probability, 2017: [1]) in which we studied Pontryagin's optimality principle for a stochastic control problem whose dynamics are given by the stochastic differential equation (SDE)

$$\begin{aligned} X_t &= x + \int_0^t b(s, X_{.\wedge s}, \mu_s^{X|Y}, u_s(Y))ds + \int_0^t \sigma(s, X_{.\wedge s}, \mu_s^{X|Y}, u_s(Y))dB_s^1, \\ Y_t &= \int_0^t h(s, X_s)ds + B_t^2,\ t \in [0,T],\ P\text{-a.s.}, \end{aligned}$$

where (B^1, B^2) is a P-Brownian motion, the controlled state process X is only observable through the observation process Y and so the control process $u = u(Y)$ is a non anticipating functional of the observation process Y. Moreover, unlike classical controlled dynamics, the coefficients σ and b do not only depend on the paths of the controlled state process X and the control $u(Y)$ but also on the law $\mu_s^{X|Y} = P \circ [E[X_s|Y_r, r \leq s]]^{-1},\ s \in [0,T]$. Motivations for such a type of dynamics are given in [1]. However, while in [1] $\mu_s^{X|Y} = P \circ [E[X_s|Y_r, r \leq s]]^{-1}$, we discuss now the case where $\mu_s^{X|Y} = P\{X_t \in (\,.\,)|Y_r, r \in [0,s]\}$ is the conditional law. The main difficulty in comparison with [1] consists in the fact that now the law $\mu_s^{X|Y}$ is not determinisitc anymore, which changes completely the techniques of the approach and makes Schauder's fixed point theorem used in [1] not applicable anymore. The main objective of the talk is to prove the weak existence and the uniqueness in law for the above dynamics.

The work is based on joint work with Juan Li and Jin Ma.