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Curvature measures and soap bubbles beyond convexity

Abstract: A fundamental result in differential geometry states that if a smooth hypersurface in a Euclidean space encloses a bounded domain and one of its mean curvature functions is constant, then it is a Euclidean sphere. This statement has been referred to as the soap bubble theorem. Major contributions are due to Alexandrov (1958) and Korevaar--Ros (1988).

While the smoothness assumption is seemingly natural at first thought, based on the notion of curvatures measures of convex bodies Schneider (1979) established a characterization of Euclidean spheres among general convex bodies by requiring that one of the curvature measures is proportional to the boundary measure. We describe extensions in two directions: (1) The role of the Euclidean ball is taken by a nice gauge body (Wulff shape) and (2) the problem is studied in a non-convex and non-smooth setting. Thus we obtain characterization results for finite unions of Wulff shapes (bubbling) within the class of mean-convex sets or even for general sets with positive reach. Several related results are established. They include the extension of the classical Steiner--Weyl tube formula to arbitrary closed sets in a uniformly convex normed vector space, formulas for the derivative of the localized volume function of a compact set and general versions of the Heintze--Karcher inequality.

(Based on joint work with Mario Santilli)