The extremals of Stanley's inequalities for partially ordered sets

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The presence of log-concave sequences is prevalent in diverse areas of mathematics ranging from geometry to combinatorics. The ubiquity of such sequences is not completely understood but the last decade has witnessed major progress towards this goal. However, we know very little about the extremals of such sequences: If we have equality somewhere along the sequence, what can be said about the sequence itself? This question is related to optimal structures (e.g. the ball in the isoperimetric inequality) and it is a necessary step towards the improvement and stability of the inequalities themselves.

I will talk about the extremals of such sequences coming from the theory of partially ordered sets (posets). R. Stanley showed in the 80's how to associate polytopes to posets and, using this correspondence (via the Alexandrov-Fenchel inequality), he proved that sequences which count the number of linear extensions of posets are log-concave. The extremals of these sequences were unknown, however, with even conjectures lacking. I will explain the resolution of this problem and the complete characterization of the extremals. The extremals turn out to be complicated and rich structures which exhibit new phenomena depending on the geometry of the associated polytopes. Towards the resolution of this problem we developed new tools that shed brighter light on the relation between the geometry of polytopes and the combinatorics of partially ordered sets.

Joint work with Zhao Yu Ma.