On Minkowski and Blaschke symmetrizations of functions and related applications

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The Minkowski symmetral of an α-concave function is defined, and some of its fundamental properties are deduced. It is shown that almost all sequences of random Minkowski symmetrizations of a quasiconcave function converge in the L^p metric ($p \geq 1$) to a spherical decreasing mean width rearrangement. A sharp extended Urysohn's type inequality for quasiconcave functions is then derived.

Using inner linearizations from convex optimization, an analogue of polytopes inscribed in a convex body is studied for log concave functions: Given a log concave function, we consider log-affine minorants defined in terms of the inner linearizations of the corresponding convex base function. As an application of the functional Minkowski symmetrizations, it is shown that any functional defined on the class of log concave functions which satisfies certain conditions is maximized by the reflectional hypo-symmetrization. As a corollary, it is shown that for every log concave function, its reflectional hypo-symmetrization is always harder to approximate by "log-affine"minorants with a fixed number of break points. This result generalizes the classical fact which states that among all convex bodies of a given mean width, a Euclidean ball is hardest to approximate in the mean width difference by inscribed polytopes with a fixed number of vertices.

The Blaschke symmetral of a function is also defined in a natural way, and some of its basic properties are proved. It is shown that under suitable assumptions, every quasiconcave function can be transformed into its symmetric decreasing surface area rearrangement by a sequence of Blaschke symmetrizations. Some applications of this result are also presented.

Some related open questions will be discussed throughout the talk, as well as very recent developments in these directions (which are joint work with Julia Novaes).