Geometric inequalities for smooth convex bodies

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In his seminal work "On simple polytopes", P. McMullen proved an array of inequalities between mixed volumes of strongly isomorphic simple polytopes that in particular broadly generalize the Alexandrov-Fenchel inequality and non-negativity of the mixed volume. Unlike these two special cases, the general inequalities do not extend to all convex bodies, as observed recently by R. van Handel.

In a joint work with Andreas Bernig and Thomas Wannerer, we prove the mixed versions of hard Lefschetz theorem and Hodge-Riemann relations for the Alesker algebra of smooth valuations on convex bodies. As a consequence, we obtain an analytic counterpart of McMullen's result, namely analogous inequalities for convex bodies with smooth boundary and positive curvature.