Maximal L^p-regularity and H^∞-calculus for block operator matrices and applications

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Many coupled evolution equations can be described via 2×2-block operator matrices of the form $\mathcal{A} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ in a product space $X = X_1 \times X_2$ with possibly unbounded entries. Here, the case of diagonally dominant block operator matrices is considered, that is, the case where the full operator $\mathcal{A}$ can be seen as a relatively bounded perturbation of its diagonal part though with possibly large relative bound. For such operators the properties of sectoriality, R-sectoriality and the boundedness of the H^∞-calculus are studied, and for these properties perturbation results for possibly large but structured perturbations are derived. Thereby, the time dependent parabolic problem associated with $\mathcal{A}$ can be analyzed in maximal L^p_t -regularity spaces, and this is illustrated by a number of applications such as different theories for liquid crystals, an artificial Stokes system, strongly damped wave and plate equations, and a Keller-Segel model. The approach developed here is based in spirit on a combination of the theory by Kalton, Kunstmann and Weis (Perturbation and interpolation theorems for the H^∞-calculus with applications to differential operators. Math. Ann., 336(4):74-801, 2006) relating R-sectoriality and the boundedness of the H^∞-calculus with concepts for diagonally dominant block operator matrices pioneered by Nagel (Towards a "matrix theory" for unbounded operator matrices. Math. Z., 201(1):57-68, 1989) for C_0-semigroups.

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