The role of w^*-convergence in the context of mild distributions

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The family of modulation spaces $M^s_{p,q}(R^d)$ shows a lot of similarities to the well-known Besov spaces and its variants. The unweighted case (i.e. the case $s = 0$) has a minimal member, namely $M_{1,1}$, also known as $S_0(R^d)$ or Feichtinger's algebra. The dual space is just the space of tempered distributions having a bounded STFT (Short-Time Fourier transform). They are called *mild distributions* nowadays. Together with the Hilbert space $L^2(R^d)$ they form a Banach Gelfand Triple (BGT), which is even well defined in the context of LCA groups.

Various BGT-isomorphisms help to understand better many of the classical questions of Fourier or Time-Frequency Analysis. In this context the w^*-convergence for (bounded) sequences of mild distributions plays an important concept. It can be used to demonstrate that "there is just one Fourier transform" (including the periodic and the discrete cases) and that any of its variants implies the concrete form of the other ones. Finally, it is argued, that this viewpoint helps to connect the continuous setting with the discrete setting, because it allows to approximate the FT of a function in $S_0(R^d)$ with the help of FFTs (fast Fourier transform).

Several recent papers by the author will be mentioned throughout the talk.