The Navier-Stokes Equations in Domains with Moving Boundaries

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Consider the instationary (Navier-)Stokes system on a family of t-dependent bounded or exterior domains, $\Omega(t)$, with moving boundary and Dirichlet boundary conditions. The main problem in the construction of local/global and even t-periodic solutions arises from a t-depending volume-preserving change of coordinates, $\phi(t)$, by which the system is transformed to a non-autonomous Stokes/Navier-Stokes system of the form

$$\partial_t u(t) + A(t)u(t) = P(t)F - P(t)u \cdot \nabla^{\phi(t)} u, \quad u(0) = u_0,$$

with a t-dependent modified Stokes operator

$$A(t) = P(t)\Big(-\Delta - \sum\nolimits_{|\alpha|\le 2} a_\alpha(x,t)\partial^\alpha + \sum\nolimits_{|\beta|\le 1} b_\beta(x,t)\partial^\beta \Big)$$

on $L^q_\sigma(\Omega_0)$ in a fixed reference domain Ω_0; here, $P(t)$ is a modified Helmholtz projection.

The analysis of this problem started with the L^2-theory by Miyakawa and Teramoto in the 80ies and local maximal regularity results in L^q of J. Saal (2006). This local theory was extended in [**?**] to global solutions for exterior domains under suitable smallness assumptions. In [2] it was shown by a uniform *BIP* property that Sobolev embedding constants and the characterization of domains of fractional powers of $A(t)$ do not depend on t. This property was the basis for the existence of mild time-periodic solutions in L^q for bounded domains with a periodically moving boundary [3]. Finally, mild solutions are shown to be regular via a Fujita-Kato iteration adapted to the non-autonomous setting [4].

In this talk we present these results and discuss first steps and new difficulties to deal with t-periodic solutions for unbounded domains [5].

The results are based on joint papers with K. Tsuda (Kyushu Sangyo University, Fukuoka), and also with H. Kozono (Waseda University, Tohoku University), D. Wegmann (TU Darmstadt).

References

[1] R. Farwig, H. Kozono, D. Wegmann: *Maximal Regularity of the Stokes Operator in an Exterior Domain with Moving Boundary and Application to the Navier-Stokes Equations.* Math. Ann. 375, 949-972 (2019)

[2] R. Farwig, K. Tsuda: *Uniform Estimates for Fractional Operators.* SN Partial Differ. Equ. Appl. 10, 2:27 (2021)

[3] R. Farwig, H. Kozono, K. Tsuda, D. Wegmann: *The Time Periodic Problem of the Navier-Stokes Equations in a Bounded Domain with Moving Boundary.* Nonlinear Analysis: Real World Applications 61, 103339 (2021)

[4] R. Farwig, K. Tsuda: *The Fujita-Kato Approach for the Navier-Stokes Equations with Moving Boundary and its Application.* J. Math. Fluid Mech. 24:77 (2022)

[5] R. Farwig, K. Tsuda: *The time periodic problem for the Navier-Stokes equations on half spaces with moving boundary. Part I: Linear theory. Part II: Nonlinear theory.* In preparation